OKLAHOMA STATE UNIVERSITY SCHOOL OF ELECTRICAL AND COMPUTER ENGINEERING



ECEN 5713 Linear Systems Spring 2001 Midterm Exam #2



DO ALL FIVE PROBLEMS

Name : ______

Student ID: _____

E-Mail Address:_____

Problem 1:

If u_1 and u_2 are linearly independent of each other, and $w_1 = au_1 + bu_2$, $w_2 = cu_1 + du_2$, please derive the relationship among $\{a, b, c, d\}$ such that w_1 and w_2 are linearly independent of each other.

Problem 2:

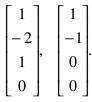
Consider the linear operator

$$A = \begin{bmatrix} 1 & 2 & -1 & 0 \\ 2 & 4 & -2 & 0 \\ 1 & 2 & -1 & 0 \end{bmatrix},$$

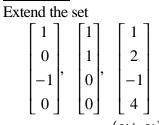
determine its rank and nullity, then find a basis for the range space and the null space of the linear operator, *A*, respectively ?

Problem 3:

Consider the subspace of \Re^4 consisting of all 4×1 column vector $x = \begin{bmatrix} x_1 & x_2 & x_3 & x_4 \end{bmatrix}^T$ with $x_1 + x_2 + x_3 = 0$. Extend the following set to form a basis for the space: $\begin{bmatrix} 1 \\ -2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 0 \\ 0 \end{bmatrix}.$



Problem 4:



to form a basis in $(\mathfrak{R}^4,\mathfrak{R})$.

Problem 5:

Let

$$V^{\perp} = Span\left(\begin{bmatrix} 1 & 1\\ 1 & -1 \end{bmatrix}, \begin{bmatrix} -5 & 1\\ 1 & 5 \end{bmatrix}, \begin{bmatrix} -1 & 2\\ 2 & 1 \end{bmatrix}\right),$$

determine the original space, V. For $x = \begin{bmatrix} 0 & 3 \\ 3 & 0 \end{bmatrix}$, find its direct sum representation of $x = x_1 \oplus x_2$,

such that $x_1 \in V$, and $x_2 \in V^{\perp}$ (I.e., the direct sum of spaces V and V^{\perp} is the set of all 2×2 matrices with real coefficients).